Buckling of Shallow Spherical Caps and Truncated Hemispheres

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Theme

BUCKLING of shallow spherical shells and truncated hemispheres subjected to nearly axisymmetric loads is studied using a digital computer program for the geometrically nonlinear analysis of arbitrarily loaded shells.[1] Small asymmetric perturbations in the load are used to disclose the minimum bifurcation buckling loads and to illuminate the imperfection-sensitivity of the shells. The pressure-loaded spherical cap was selected because previous results had indicated that it exhibits all of the traits desired: its axisymmetric behavior is snap-buckling, bifurcation loads exist below the axisymmetric snap-buckling loads, and it is sensitive to imperfections. The truncated hemisphere under axial tension was selected because the experimental buckling loads are approximately one-half of the theoretical bifurcation loads.[2] Imperfection-sensitivity was suspected to cause the discrepancy.

Content

In the analysis, all dependent variables are expressed in trigonometric series with the argument $n\theta$, where θ is the circumferential coordinate. The nonlinear terms in the governing equations are products of series that are expanded into single series. The coefficients of like trigonometric arguments are then grouped to form coupled sets of equations. The sets are uncoupled by treating the nonlinear terms as known quantities or pseudoloads. A finite-difference formulation is employed for the meridional derivatives of the variables. This leads to one set of algebraic equations for each value of n considered. The sets are repeatedly solved, using an elimination procedure, until the solution converges.

A load-displacement history is obtained by incrementing the applied loads. The load increment is reduced if the solution fails to converge after a specified number of iterations. The value of the load at the final convergence failure is called the final load. Since the method of solution is based on the pseudoload approach, the existence of a maximum or an inflection point on a softening load-deflection curve represents a type of behavior for which a solution can be obtained only below the point of zero or nearly zero slope. The existence of a stiffening behavior can also cause a convergence failure whenever the slope becomes too steep. A typical execution time using 40 stations and 2 modes is 30 sec using the FORTRAN IV, Level H Compiler, OPT = 2. A copy of the program for static analysis and a Users' Manual can be obtained through COSMIC‡ (Computer Program Abstracts Accession Number M70-10098, LAR-10736). A program with dynamic analysis capability will be available in the latter half of 1971.

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The geometry of the spherical cap is specified by the nondimensional geometric parameter $\lambda = 2[3(1-\nu^2)]^{1/4}(H/h)^{1/2}$, where ν is Poisson's ratio, H is the rise, and h is the thickness. The nearly axisymmetric uniform pressure load applied to the shell is given by $q(1+\varepsilon\cos N\theta)$, where ε is a small number and N is a particular value of n. The buckling pressure for a complete sphere is q_0. Approximate bifurcation loads were obtained at several values of λ using a two-mode solution with $0.0001 \leqq \varepsilon \leqq 0.0005$. The asymmetric displacements were negligible until the applied load reached $\sim 95\%$ of the final load. Failure of the solution to converge was attributed to the softening nonlinearities in the asymmetric harmonic. Thus, the conditions for a bifurcation analysis were satisfied. The asymptotic bifurcation loads were in very good agreement with previously published results.

An imperfection-sensitive shell is one for which the buckling load is reduced due to the presence of asymmetric imperfections.[3] Figure 1 presents the load-displacement curves for $W^{(0)}$, the normal displacement in the axisymmetric harmonic, at station 24 (a total of 40 stations were used) for $\lambda = 8$ and several values of ε. Also shown is the initial slope of the bifurcation branch of the equilibrium path predicted by Fitch and Budiansky[4] using Koiter's initial postbuckling theory. In Refs. 3–5, the slope of the equilibrium path is related to the degree of imperfection sensitivity. The backward sloping equilibrium path of Fig. 1 shows this pressure-loaded cap to be very sensitive to imperfections.[3-5]

Yao[2] reported theoretical bifurcation buckling loads and experimental buckling loads for two clamped, truncated hemispherical shells (A and B) subjected to axial tension forces at each edge. Yao's bifurcation analysis for the buckling loads was based upon a linear prebuckling axisymmetric solution. Consequently, asymptotic bifurcation loads were obtained for shell B using both a linear and a nonlinear axisymmetric analysis. Asymptotic bifurcation loads based on a nonlinear axisymmetric analysis could not be obtained for shell A because of a convergence failure in the axisymmetric harmonic at a load below the mini-

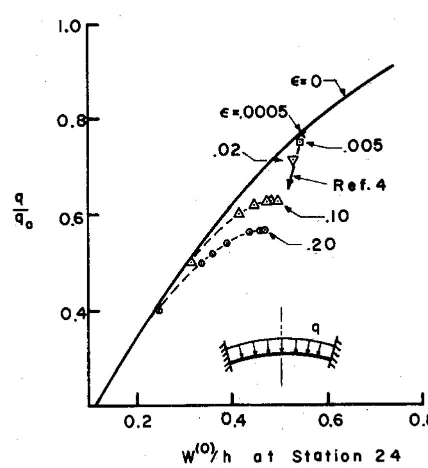

Fig. 1 Load-axisymmetric displacement curve for $\lambda = 8$ with several values of ε.

mum bifurcation load due to a nonlinear stiffening behavior. The present results using 40 stations along the meridian and a linear axisymmetric analysis agree well with Yao's results for $N \geqq 30$, (N critical ≈ 40), but for $N < 30$, the solution in the asymmetric harmonic failed to converge because of stiffening in the region near the truncated edge instead of softening. Similar results were obtained for shell B where the results were good only for $N \geqq 40$ ($N_{\text{critical}} \approx 70$).

The imperfection-sensitivity of shell A could not be established, since bifurcation loads could be determined only for the linear axisymmetric analysis. Hence, it was not possible to assess the effect the asymmetric harmonic has upon the axisymmetric harmonic. The sensitivity of shell B is indicated in Fig. 2, where P_0 is the meridional membrane force at the equator, and E is the elastic modulus. Simulating the asymmetric imperfections with a circumferential variation in the axial load localized the imperfections near the edges and did not reveal the sensitivity of the shell. Consequently, the imperfections were simulated by a very small uniform pressure in the Nth harmonic, as in the case of the pressure-loaded spherical cap.

Hutchinson,[3,5] in a study of the initial postbuckling behavior of torodial shells based on Koiter's theory, suggested that the imperfection sensitivity of Yao's clamped truncated hemispheres should agree qualitatively with the degree of sensitivity he obtained for the simply supported hemispherical segments subjected to axial tension. Based on his prediction, shell B is not as sensitive to imperfections as the axially-loaded cylinder or the pressure-loaded spherical shell, but it does fall in a region where a significant reduction in the buckling load may occur. A plot of the initial postbuckling slope of the generalized load-axial deflection curve given in Ref. 5 for the shell corresponding to shell B is shown in Fig. 2. For $\varepsilon \leqq 0.01$, the final load is slightly below the predicted bifurcation load, but the final displacements in the axisymmetric mode are increased, in agreement with Hutchinson's predicted postbuckling path. Thus, the shell appears to be sensitive to small imperfections, but the behavior differs from that of the spherical cap since the final axisymmetric displacement increases with increasing imperfection instead of decreasing. For larger imperfections the asymmetric mode had a pronounced effect upon the axisymmetric behavior in the central region of the shell, and there are significant reductions in the value of the final load with virtually no change in the final value of the axisymmetric displacements. This type of behavior for the larger imperfections is similar to that of the spherical cap shown in Fig. 1. Furthermore, the experimental buckling loads of shells A and B are approximately one half of the theoretical bifurcation loads. This is a common reduction factor for spherical shells. Thus, the degree of imperfection-sensitivity of the truncated hemispheres may be more closely related to the results for the larger ε than to the initial postbuckling behavior.

The two problems considered here were solved using only two circumferential harmonics; the axisymmetric and one asymmetric. An alternative method of solution would be to use finite-element theory or finite differences for all of the derivatives. However, when the critical harmonic is very large, as is the case for the two truncated hemispheres where the critical harmonics were 40 and 70, a very large number of elements or meridional mesh lines would be required for adequate resolution. This would lead to a very large number of equations to be solved. Thus, when only a few harmonics dominate the response the method discussed here is considerably more efficient.

Additional imperfection-sensitivity studies are presently being carried out using actual deviations in the shape of the shell as the asymmetric perturbations.

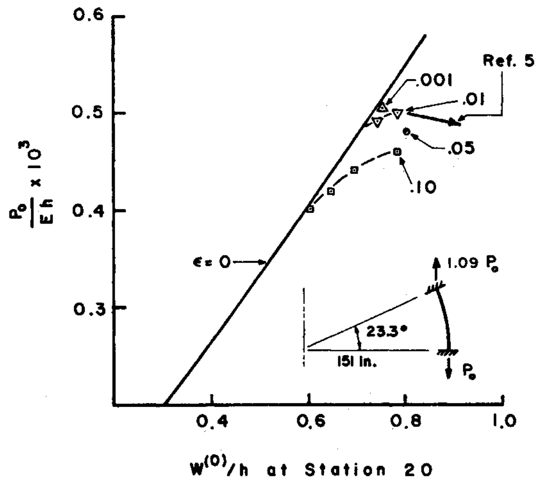

Fig. 2 Load-axisymmetric displacement curve for shell B with several values of ε.

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